Pre-Calculus CP 1 – Section 3.5 Notes **Name:** _____
EVEN MORE Applications of Exponential and Logarithmic Functions

Half Life

When examining half-life, the important thing to remember is that half-life represents the amount of time for half of the substance to disappear.

You will often use the Law of Uninhibited Growth and Decay when doing half-life problems:

$$A(t) = A_0 e^{kt}$$

Examples

1) Iodine 131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$ where A_0 is the initial amount present and A is the amount present at time t (in days). What is the half-life of Iodine 131?

2) The half-life of radium is 1690 years. If 10 grams is present now, how much will be present in 50 years?

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Newton's Law of Cooling

The temperature, u, of a heated object at a given time, t, can be modeled by the following function:

$$u(t) = T + (u_0 - T)e^{kt},\ k < 0$$

where T is the constant temperature of the surrounding medium, u_0 is the initial temperature of the heated object, and k is a negative constant.

Example

An object is heated to 100 degrees Celsius and is then allowed to cool in a room whose air temperature is 30 degrees Celsius.

a) If the temperature of the object is 80°C after 5 minutes, when will the temperature be 50°C?

b) Determine the elapsed time before the temperature of the object is 35°C.

c) What do you notice about $u(t)$, the temperature, as time passes?

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Logistic Growth Model

This is exponential growth, but with inhibitions…in other words, certain factors will *limit* the growth:

$$P(t)=\frac{c}{1+ae^{-bt}},\ b>0\ \&\ c>0$$

where a, b, and c are constants. c is called the *carrying capacity* because it is the ceiling for how big this value can become.

Example

Fruit flies are placed in a half-pint milk bottle with a banana and yeast plants. Suppose that the fruit fly population after t days is given by $P(t)=\frac{230}{1+56.5e^{-0.37t}}$.

a) What is the carrying capacity of the half-pint bottle?

b) How many fruit flies were initially placed in the half-pint bottle?

c) When will the population of fruit flies be 180?